

NAG Toolbox for MATLAB

f08ne

1 Purpose

f08ne reduces a real general matrix to Hessenberg form.

2 Syntax

```
[a, tau, info] = f08ne(ilo, ihi, a, 'n', n)
```

3 Description

f08ne reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation: $A = QHQ^T$.

The matrix Q is not formed explicitly, but is represented as a product of elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

The function can take advantage of a previous call to f08nh, which may produce a matrix with the structure:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{pmatrix}$$

where A_{11} and A_{33} are upper triangular. If so, only the central diagonal block A_{22} , in rows and columns i_{lo} to i_{hi} , needs to be reduced to Hessenberg form (the blocks A_{12} and A_{23} will also be affected by the reduction). Therefore the values of i_{lo} and i_{hi} determined by f08nh can be supplied to the function directly. If f08nh has not previously been called however, then i_{lo} must be set to 1 and i_{hi} to n .

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **ilo** – int32 scalar

2: **ihi** – int32 scalar

If A has been output by f08nh, then **ilo** and **ihi** must contain the values returned by that function. Otherwise, **ilo** must be set to 1 and **ihi** to n .

Constraints:

if $n > 0$, $1 \leq \text{ilo} \leq \text{ihi} \leq n$;
if $n = 0$, **ilo** = 1 and **ihi** = 0.

3: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

The n by n general matrix A .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **a**.

n, the order of the matrix *A*.

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

a contains the upper Hessenberg matrix *H* and details of the orthogonal matrix *Q*.

2: **tau(*)** – double array

Note: the dimension of the array **tau** must be at least $\max(1, n - 1)$.

Further details of the orthogonal matrix *Q*.

3: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter *i* had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **ilo**, 3: **ihi**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed Hessenberg matrix *H* is exactly similar to a nearby matrix $(A + E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of *n*, and ϵ is the *machine precision*.

The elements of *H* themselves may be sensitive to small perturbations in *A* or to rounding errors in the computation, but this does not affect the stability of the eigenvalues, eigenvectors or Schur factorization.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}q^2(2q + 3n)$, where $q = i_{hi} - i_{lo}$; if $i_{lo} = 1$ and $i_{hi} = n$, the number is approximately $\frac{10}{3}n^3$.

To form the orthogonal matrix Q f08ne may be followed by a call to f08nf:

```
[a, info] = f08nf(ilo, ihi, a, tau);
```

To apply Q to an m by n real matrix C f08ng may be followed by a call to f08ng. For example,

```
[c, info] = f08ng('Left', 'No Transpose', ilo, ihi, a, tau, c);
```

forms the matrix product QC .

The complex analogue of this function is f08ns.

9 Example

```
ilo = int32(1);
ihi = int32(4);
a = [0.35, 0.45, -0.14, -0.17;
     0.09, 0.070000000000000001, -0.54, 0.35;
     -0.44, -0.33, -0.03, 0.17;
     0.25, -0.32, -0.13, 0.11];
[aOut, tau, info] = f08ne(ilo, ihi, a)
```

```
aOut =
    0.3500    -0.1160    -0.3886    -0.2942
   -0.5140     0.1225     0.1004     0.1126
   -0.7285     0.6443    -0.1357    -0.0977
    0.4139    -0.1665     0.4262     0.1632
tau =
    1.1751
    1.9460
         0
info =
         0
```